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A logistic First Order Difference Equation of Periodic Chemotherapy Model

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ABSTRACT

A logistic difference equation behavior with a periodic chemetherapy model is used to growth of cancer cells. To study the growth rate of behavior of the chemotherapy

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INTRODUCTION

The logistic growth model has been used as basic model of cell growth and tumor cell growth . The same model [9], which a deteriorating environment through the use of decreasing growth rates and carrying capacities. In this Model to study the periodic forms of the growth rate of periodic chemotherapy. [3-8].

A LOGISTIC DIFFERENCE EQUATION MODEL

The Logistic difference equation model is modified so that there is rate of growth to taken into account chemotherapy. The general form is

$$y(n) = y(n)\left[r\left((1-b(n)) - \frac{y(n)}{k}\right) + 1\right] \qquad (1) \qquad 1 \le \mathrm{n} \le \infty$$

Where $y(n)$ the cell is mass, r is the growth rate, k is the carrying capacity and $b(n)$ is a periodic function representing the cell mass of chemotherapeutic effects. If $b(n) \equiv 0$, the there are no chemotherapeutic effects and equilibrium is K, if $b(n) \equiv b < 1$, the equilibrium is $K(1-b)$. Conversely the equilibrium is 0 if $b(n) \equiv b > 1$. If $\left[1-b(n)\right]$ is positive for all t, then there is tumor growth with a growth rate reduced and there will be equilibrium between zero and K. The equation (1) becomes reduced to a simpler form, putting $y(n) = Ku(n)$.

$$\mathrm{u}(n) = u(n)\left[r\left((1-b(n)) - u(n)\right) + 1\right] \qquad (2)$$

The function $b(n)$ can take on various periodic forms

$$b(n) = \begin{Bmatrix} b, & n_1\sigma \le n \le a + n_1\sigma, \\ 0, & a + n_1\sigma \le n \le (n_1+1)\sigma \end{Bmatrix} \qquad (3)$$

or exponentially decaying piecewise periodic functions

$$b(n) = b\exp a(n - n_1\sigma), \qquad n_1\sigma \le n \le (n_1+1)\sigma \qquad (4)$$

or the periodic function

$$b(n) = b\sin\left(\frac{2\pi n}{\sigma}\right) \qquad (5)$$

SOLUTION OF A LOGISTICS DIFFERENC EQUATION MODEL

The equation (2) is solved by various methods for $b(n)$. The equation is Bernoulli type and can be solved exactly.

$$u(n) = \frac{u_0 \exp r \sum_{n_1}^{n} [1 - b(n)]}{1 + \left(\frac{u_0}{r}\right) \sum_{n_1}^{n} \exp r \sum_{n}^{s} [1 - b(\mathrm{n}_2)]} \quad (6)$$

Using this solution and $b(n)$ is periodic. The equation (6) describes the growth of the tissue over each period where u_0 is the initial cell mass of the period. The difference equation is

$$u(n+1)\sigma = \frac{u_{n_1\sigma} \exp r \sum_{n_1\sigma}^{(n+1)\sigma} [1 - b(n)]}{1 + \left(\frac{u_{n_1\sigma}}{r}\right) \sum_{n_1\sigma}^{(n+1)\sigma} \exp r \sum_{n}^{s} [1 - b(\mathrm{n}_2)]} \quad (7)$$

The stable equilibrium of this difference equation. Solving the equation

$$u_{Eq} = \frac{u_{Eq} \exp r \sum_{n_1\sigma}^{(n+1)\sigma} [1 - b(n)]}{1 + \left(\frac{u_{Eq}}{r}\right) \sum_{n_1\sigma}^{(n+1)\sigma} \exp r \sum_{n}^{s} [1 - b(\mathrm{n}_2)]} \quad (8)$$

For u_{Eq}, we can determine the equilibria. They are

$$u_{Eq} = 0 \quad (9)$$

$$u_{Eq} = \frac{r\left(\exp r \sum_{n_1\sigma}^{(n+1)\sigma} [1 - b(n)] - 1\right)}{\sum_{n_1\sigma}^{(n+1)\sigma} \exp r \sum_{n}^{s} [1 - b(\mathrm{n}_2)]} \quad (10)$$

Equation (10) is equation to zero for $\langle b(n) \rangle = 1$ which is the bifurcation from a positive stable equilibrium to zero stable equilibrium. That is for $0 \le \langle b(n) \rangle \le 1$, equilibrium (10) is stable and equilibrium (9) is unstable. For $\langle b(n) \rangle > 1$ the stability switches and equilibrium (9) becomes stable while equilibrium (10) switches to unstable. Therefore, the cells have a zero equilibrium when

$$\langle b(n) \rangle > 1 \quad (11)$$

In the above, we define

$$\langle b(n) \rangle \equiv \frac{1}{\sigma} \sum_{0}^{r} b(n) \quad (12)$$

as the mean value of $b(n)$

Now we examine the step function form of $b(n)$ (equation (3)) directly by examining the solution over each piece of the period σ.

$$u(n)=\begin{cases}\dfrac{(1-b)u_{n_1\sigma}}{u_{n_1\sigma}+\left[(1-b)-u_{n_1\sigma}\right]\exp-(1-b)r(n-n_1\sigma)}, & n_1\sigma\le n\le a+n\sigma,\\[2ex] \dfrac{u_{(a+n_1\sigma)}}{u_{(a+n_1\sigma)}+\left[(1-b)-u_{(a+n_1\sigma)}\right]\exp-r(\mathrm{n}-(\mathrm{a}+n_1\sigma))}, & a+n\sigma\le n\le(\mathrm{n}+1)\sigma,\end{cases}\quad(13)$$

The two solutions are $a+n_1\sigma$.

$$u(n+1)\sigma=\frac{(1-b)u_{n_1\sigma}}{u_{n_1\sigma}+\left[(1-b)-u_{n_1\sigma}\right]\exp-(1-b)\,\mathrm{a}\,r}, \quad (14)$$

From this solution, the difference equation that relates the size of $u(n)$ at the one period $(u_{n_1\sigma})$ to next period $(u_{(n_1+1)\sigma})$. The Poincare map for the equation (13) becomes

$$u_{(n_1+1)\sigma}=\frac{1}{1+\left[\dfrac{u_{n_1\sigma}+\left[(1-b)-u_{n_1\sigma}\right]\exp-(1-b)ar}{(1-b)u_{n_1\sigma}}-1\right]\exp-r(\sigma-a)} \quad (15)$$

The equilibria for this difference equation

$$u_{Eq}=0 \quad (16)$$

$$u_{Eq}=\frac{1-\exp r(ab-\sigma)}{1-\left(\dfrac{1}{(1-b)}\right)\left(\exp-ar(1-b)-\mathrm{b}\right)\exp-r(\sigma-a)} \quad (17)$$

The Special case of equations (9) and (10) where the bifurcation from equilibrium (17) being stable to equilibrium (16) be stable is $ab=\sigma$

CONCLUSION

This model is to more accurately model of chemotherapy. A few possibilities are varying the capacity K to model either the tumor bed effect [4,11]. This model gives the bifurcation between reduced steady state cell survival and cell destruction. It can be the basis for studying the chemotherapeutic effects on both cancer cell tissue and normal cell tissue such bone marrow.

REFERENCE

1. Panetta. A logistic model of periodic chemotherapy, Appl. Math. Lett 1995; 8 (4): 83-86.

2. J.R. Usher, Some mathematical models for cancer chemotherapy. Computers Math. Applic. 199428 (9):73-80.
3. H.E. Skipper. On mathematical modeling of critical variables in cancer treatment goals: Better understanding of the past and better planning in the future. Bull. Math. Biol. 1986; 48 (3/4):253-278.
4. J.C. Panetta and J.A. Adam, A mathematical model of cycle-specific chemotherapy. Mathl. Comput. Modeling 1995;22 (2):67-82 .
5. S. Michelson, B.E. Miller, A.S. Glicksman, J.T. Leith, Tumor micro-ecology and competitive interactions, Y. Theoret. Biol. 1987; 128 (2):233-246.
6. S. Michelson, J.T. Leith, Effects of differential cell kill on the dynamic composition of heterogeneous tumors, Computers Math. Applic. 1990; 20 (4-6):149-159.
7. S. Michelson, J.T. Leith, Tumor heterogeneity: A review of the theory, Drug News 8J Perspectives 1993; 6 (9):655-661.
8. T.E. Wheldon, Mathematical Models in Cancer Research**,** Adam Hilger, Bristol, UK, 1988.
9. G.G. Steel, Growth Kinetics of Tumours, Clarendon, Oxford, 1977.
10. B. Drewinko, M. Patchen, Li-Y. Yang and B. Barlogie, Differential Killing Efficacy of Twenty Antitumor Drugs on Proliferating and Non proliferating Human Tumor Cells Cancer Res 1981;41:2328-2333.